
AP CALCULUS AB

SUMMER PACKET

This packet is intended to prepare you for AP Calculus AB by reviewing prerequisite algebra and pre-calculus skills and covering the first chapter calculus textbook. (With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter over the summer.)

It is due on the first day of school and worth practice points based on completion. At the end of the first week of school, there will be a Summer Packet quiz covering this review content.

The packet is lengthy, so please start early! While many of the exercises cover basic algebra skills, you will encounter a few tough exercises. Embedded in this packet are short videos to guide you if you get stuck. If you need further assistance, please email me and we can schedule a meeting in-person or via Zoom. (stephen.kemp@district196.org)

Have a wonderful and relaxing summer! I am looking forward to delving into the study and exploration of a branch of mathematics which has been referred to as "one of the supreme accomplishments of the human intellect."

Calculus truly is a fascinating course—you will love it!

— Mr. Kemp



The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.



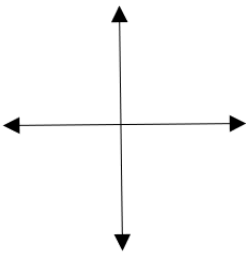
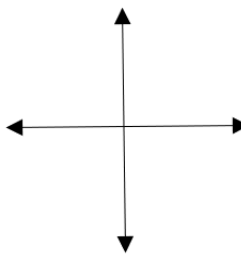
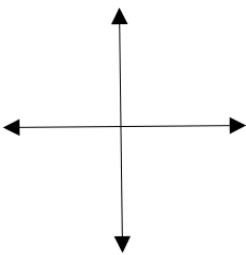
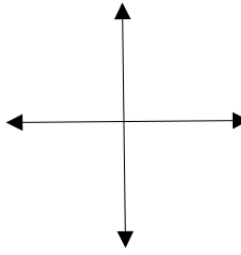
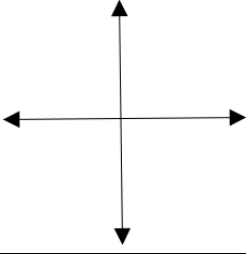
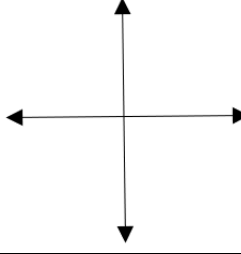
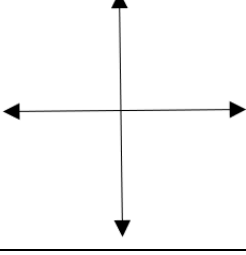
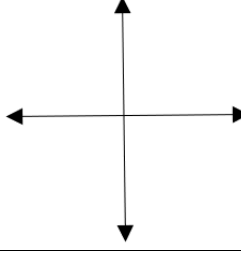
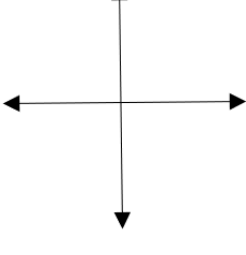
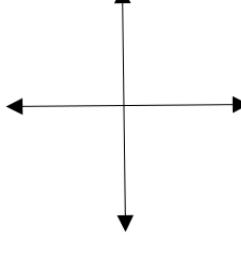
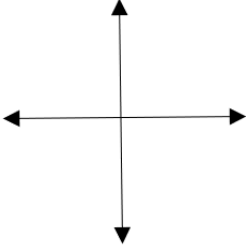
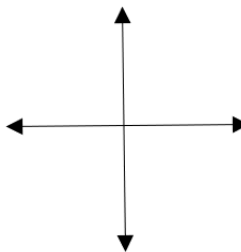
<p>LINES</p> <p>Slope-intercept: $y = mx + b$</p> <p>Point-slope: $y - y_1 = m(x - x_1)$</p> <p>Standard: $Ax + By = C$</p> <p>Horizontal line: $y = b$ (slope = 0)</p> <p>Vertical line: $x = a$ (slope = undefined)</p> <p>Parallel \rightarrow same slope</p> <p>Perpendicular \rightarrow opposite reciprocal slopes</p>	<p>QUADRATICS</p> <p>Standard: $y = ax^2 + bx + c$</p> <p>Vertex: $y = a(x - h)^2 + k$</p> <p>Intercept: $y = a(x - p)(x - q)$</p> <p>Parabola opens: up if $a > 0$ down if $a < 0$</p> <p>Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p>
<p>EXPONENTIAL PROPERTIES</p> <p>$x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$</p> <p>$\frac{x^a}{x^b} = x^{a-b}$ $\sqrt[n]{x^m} = x^{m/n}$</p> <p>$x^0 = 1$ ($x \neq 0$) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$</p> <p>$x^{-n} = \frac{1}{x^n}$ In general, it is fine to have negative exponents in your answers!</p>	<p>LOGARITHMS</p> <p>$y = \log_a x$ is equivalent to $a^y = x$</p> <p>$\log_b(mn) = \log_b m + \log_b n$</p> <p>$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$</p> <p>$\log_b(m^p) = p \log_b m$</p>
<p>TRIGONOMETRIC IDENTITIES</p> <p>$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$ $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$</p> <p>$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$</p> <p>$\sin(2x) = 2 \sin x \cos x$ $\cos(2x) = \cos^2 x - \sin^2 x$ or $1 - 2 \sin^2 x$ or $2 \cos^2 x - 1$</p>	

Whenever you see a video icon, click it to watch a short video about the content. For example, this video link will help you with the next page.



You are expected to know the general shape, domain, and range of each parent function in the table.

"Parent" functions mean no transformations have been applied.
Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

<p>Linear functions (x)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $(-\infty, \infty)$</p> 	<p>Quadratic functions (x^2)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $[0, \infty)$</p> 
<p>Odd-degree polynomials (x^n for odd n)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $(-\infty, \infty)$</p> 	<p>Even-degree polynomials (x^n for even n)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $[0, \infty)$</p> 
<p>Exponential functions (a^x, e^x)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $(0, \infty)$</p> <p>Horizontal asymptote: $y = 0$</p> 	<p>Logarithmic functions ($\log_a x, \ln x$)</p> <p>D: $(0, \infty)$</p> <p>R: $(-\infty, \infty)$</p> <p>Vertical asymptote: $x = 0$</p> 
<p>Sinusoidal functions ($a \sin x, a \cos x$)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $[-a, a]$</p> 	<p>Tangent function ($\tan x$)</p> <p>D: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$</p> <p>R: $(-\infty, \infty)$</p> <p>Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$</p> 
<p>Reciprocal functions ($\frac{1}{x}$)</p> <p>D: $(-\infty, 0) \cup (0, \infty)$</p> <p>R: $(-\infty, 0) \cup (0, \infty)$</p> <p>Vertical asymptote: $x = 0$</p> <p>Horizontal asymptote: $y = 0$</p> 	<p>Absolute value functions (x)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $[0, \infty)$</p> 
<p>Even root functions ($\sqrt[n]{x}$, $\sqrt[n]{x}$ for even n)</p> <p>D: $[0, \infty)$</p> <p>R: $[0, \infty)$</p> 	<p>Odd root functions ($\sqrt[n]{x}$, $\sqrt[n]{x}$ for odd $n > 1$)</p> <p>D: $(-\infty, \infty)$</p> <p>R: $(-\infty, \infty)$</p> 

For #1-8, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

2. Crossing the x -axis at $x = -3$ and the y -axis at $y = 6$

3. Containing the points $(-6, -1)$ and $(3, 2)$

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

5. Write an equation of a line passing through $(-4, 2)$ with a slope of 0.

6. Write an equation of a line passing through $(2, 8)$ that is parallel to $y = \frac{5}{6}x - 1$.

7. Write an equation of a line passing through $(4, 7)$ that is perpendicular to the y -axis.

8. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

For #9-16, solve each equation for x . Note that some equations will have a specific value, but most will have a solution in terms of other variables. (For example: $x = \frac{a+b}{c}$ may be a solution.)

9. $x^2 + 3x = 8x - 6$

10. $\frac{2x-5}{x+y} = 3 - y$

$$11. 3xy + 6x - xz = 12$$

$$12. A = ax + bx$$

$$13. cx = vx$$

$$14. r = t - x(z - y)$$

$$15. \frac{3+x}{5-x} = 6 + y$$

$$16. \frac{y+2}{4-x} = 4(2 - z)$$

For #17-22, solve each quadratic by factoring.



This video demonstrates factoring. For the exercises below, you must factor and then solve.

$$17. x^2 - 4x - 12 = 0$$

$$18. x^2 - 6x + 9 = 0$$

$$19. x^2 - 9x + 14 = 0$$

20. $x^2 - 36 = 0$

21. $9x^2 - 1 = 0$

22. $4x^2 + 4x + 1 = 0$

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$

24. $g(-2) =$

25. $f(1) + g(0) =$

26. $f(0) \cdot g(0) =$

27. $\frac{g(-6)}{f(-6)} =$

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f(g(x)) =$

29. $g(f(x)) =$

30. $f(f(4)) =$

31. $g(h(-4)) =$

32. $f(g(h(1))) =$

$$33. f(g(x-1)) =$$

$$34. g(f(x^3)) =$$

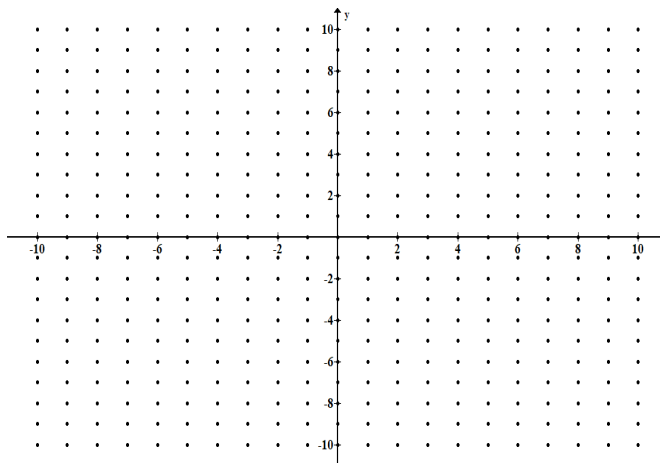
$$35. \frac{f(x+h)-f(x)}{h} =$$



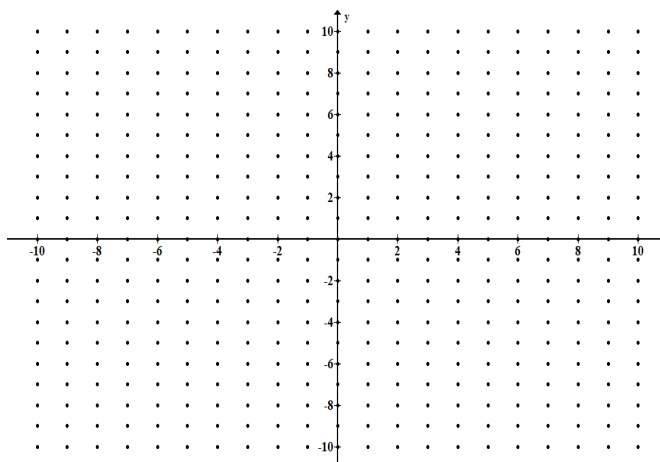
This is an important for calculus.
What is the name of this expression?

For #36-38, graph each piecewise function.

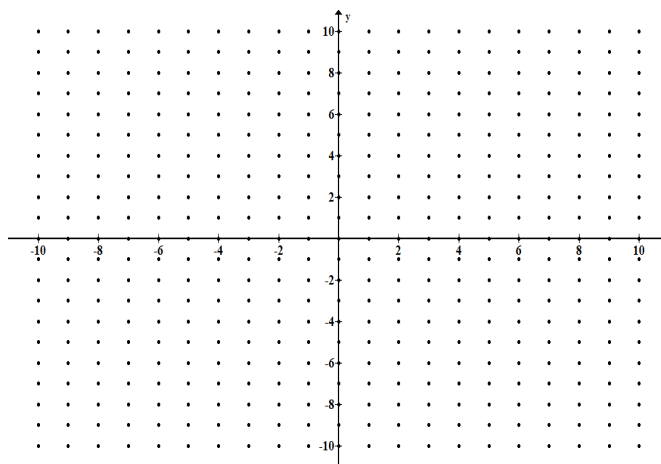
$$36. f(x) = \begin{cases} x+3 & ; x < 0 \\ -2x+5 & ; x \geq 0 \end{cases}$$



$$37. g(x) = \begin{cases} \frac{1}{2}x & ; -4 \leq x \leq 2 \\ 2x-3 & ; x > 2 \end{cases}$$



$$38. h(x) = \begin{cases} |x| & ; x \leq 1 \\ 2 - |x - 2| & ; x > 1 \end{cases}$$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.



$$39. 5^x = \frac{1}{5}$$

$$40. 6^x = 1296$$

$$41. 6^{2x-7} = 216$$

$$42. 5^{3x-1} = 49$$

$$43. 10^{x+5} = 125$$

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

$$44. e^{\ln 4} =$$

$$45. e^{2 \ln 3} =$$

$$46. \ln e^9 =$$

$$47. 5 \ln e^3 =$$

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$

49. $3e^x = 120$

50. $e^x - 8 = 51$

51. $\ln x = 2.5$

52. $\ln(3x - 2) = 2.8$

53. $2 \ln(e^x) = 5$

For #54-66, find the exact value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!



54. $\sin 120^\circ =$ _____

55. $\cos \frac{11\pi}{6} =$ _____

56. $\tan 225^\circ =$ _____

57. $\sin\left(-\frac{2\pi}{3}\right) =$ _____

58. $\sin 150^\circ =$ _____

59. $\tan \frac{7\pi}{4} =$ _____

60. $\csc\left(\frac{\pi}{4}\right) =$ _____

61. $\sec(-210^\circ) =$ _____

62. $\cot\left(\frac{5\pi}{4}\right) =$ _____

63. $\sin\left(\frac{9\pi}{4}\right) =$ _____

64. $\sec\left(-\frac{\pi}{4}\right) =$ _____

65. $\tan\left(-\frac{4\pi}{3}\right) =$ _____

66. $\cos\left(\frac{8\pi}{3}\right) =$ _____

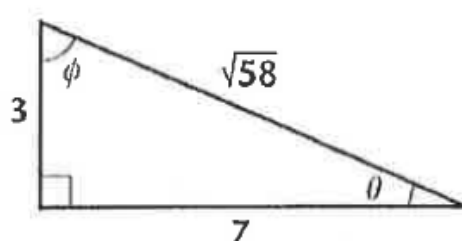
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do NOT need to rationalize the denominator.

67. $\sin \theta =$ _____

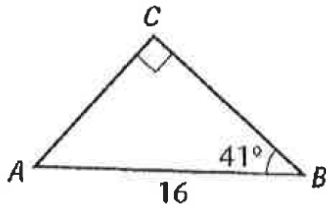
68. $\cos \theta =$ _____

69. $\tan \phi =$ _____

70. $\sec \phi =$ _____



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$$m\angle A = \underline{\hspace{2cm}}$$

$$AC = \underline{\hspace{2cm}}$$

$$CB = \underline{\hspace{2cm}}$$

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.



$$72. \sin^{-1}\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$$

$$76. \tan^{-1}(-1) = \underline{\hspace{2cm}}$$

$$73. \sin^{-1}(-1) = \underline{\hspace{2cm}}$$

$$77. \tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \underline{\hspace{2cm}}$$

$$74. \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \underline{\hspace{2cm}}$$

$$78. \sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) = \underline{\hspace{2cm}}$$

$$75. \tan^{-1}(\sqrt{3}) = \underline{\hspace{2cm}}$$

$$79. \sin^{-1}(\cos(0)) = \underline{\hspace{2cm}}$$

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

For #81-83, find the inverse of each function.

$$81. g(x) = \frac{5}{x-2}$$

$$g^{-1}(x) = \underline{\hspace{2cm}}$$

$$82. f(x) = \frac{x^2}{3}$$

$$f^{-1}(x) = \underline{\hspace{2cm}}$$

$$83. y = \sqrt{4-x} + 1$$

$$y^{-1} = \underline{\hspace{2cm}}$$

84. If the graph of $f(x)$ has the point $(2,7)$, then
what is one point on the graph of $f^{-1}(x)$? _____

For #85-89, write each inequality in interval notation. For example, $x > 3$ becomes $(3, \infty)$.

85. $1 < x \leq 10$ _____

86. $x < 0$ **or** $x \geq 4$ _____

87. $x \geq -2$ _____

88. $x \geq 4$ **and** $x > 10$ _____

89. $x > 5$ **or** $x < 7$ _____

For #90-99, find the domain and range of each function. Write answers in interval notation. Confirm your answer by graphing the function on your calculator. The parent functions on page 3 of this packet will help.



90. $f(x) = \sqrt{x+5}$

D: _____

R: _____

91. $g(x) = x^2 - 5$

D: _____

R: _____

92. $y(t) = \frac{1}{t+7}$

D: _____

R: _____

93. $h(x) = \frac{5}{x^2+1}$

D: _____

R: _____

94. $f(x) = \sqrt{x^2+5}$

D: _____

R: _____

95. $g(t) = t^3 + 2t - 7$

D: _____

R: _____

96. $h(x) = 3 \sin(\pi x) - 1$

D: _____

R: _____

97. $y(x) = \sqrt[5]{2x+3}$

D: _____

R: _____

98. $f(x) = -3e^{2x} + 5$

D: _____

R: _____

99. $g(t) = \log_4(x-2) + 1$

D: _____

R: _____

For #100-102, find the difference quotient of each function. (Refer back to #35 if needed.)

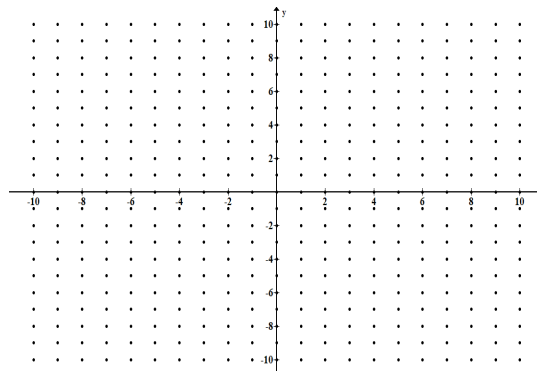
100. $g(x) = x^2 - 3x$

101. $f(x) = \frac{2}{x+1}$

102. $h(x) = \sqrt{x-3}$

The remaining exercises are more challenging and specifically from concepts and skills covered in Pre-Calculus. You must show all work to earn credit.

103. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$



D: _____ R: _____

104. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.



- a. Use your calculator to find the relative maximum and minimum y-value of $f(x)$.

min = _____ max = _____

- b. State the domain of $f(x)$ in interval notation.

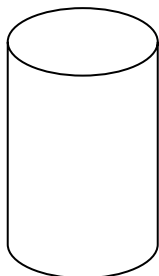
D: _____

- c. State when the function is increasing and decreasing. Write in interval notation.

increasing: _____ decreasing: _____

105. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Show all work to earn credit.

106. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?

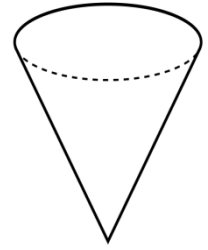


$r =$ _____ $h =$ _____

107. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water. Write an expression for the volume of the water in terms of its...

a. radius

Use similar triangles



$$V(r) = \underline{\hspace{2cm}}$$

b. height

$$V(h) = \underline{\hspace{2cm}}$$

108. Evaluate the following limits algebraically.



a. $\lim_{x \rightarrow 1} e^{x^3 - x} =$

b. $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3} =$

c. $\lim_{x \rightarrow 5^+} \frac{x + 5}{x - 5} =$

d. $\lim_{h \rightarrow 0} \frac{(h - 1)^3 + 1}{h} =$

e. $\lim_{x \rightarrow 0} \frac{\sqrt{4 + x} - 2}{x} =$

f. For constant c , $\lim_{x \rightarrow c} x =$

g. For constants a and c , $\lim_{x \rightarrow a} c =$

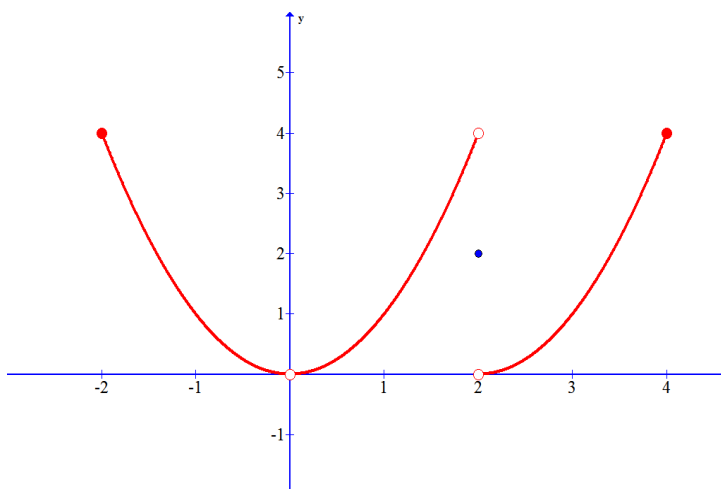
h. $\lim_{h \rightarrow 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} =$

i. $\lim_{x \rightarrow \infty} \frac{3x^3 + 5x^2 - 7x}{8x^3 - 13} =$

j. $\lim_{v \rightarrow 4^+} \frac{4-v}{|4-v|} =$

k. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} =$

109. Use the graph of the function to answer the following questions. Be as specific as possible



a. $f(2) =$ _____

b. $\lim_{x \rightarrow 2^+} f(x) =$ _____

c. $\lim_{x \rightarrow 2^-} f(x) =$ _____

d. $\lim_{x \rightarrow 2} f(x) =$ _____

e. $f(0) =$ _____

f. $\lim_{x \rightarrow 0} f(x) =$ _____

110. Find the instantaneous rate of change at any point x for the function $f(x) = 2x^2 - x$ using the definition of the derivative below.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

111. Estimate the area under the curve $y = x^2 + 1$ on the interval $[0,2]$ using Riemann sums with four equally-spaced subdivisions with heights determined by using:



a. **LRAM (left endpoint)**

b. **RRAM (right endpoint)**

c. **MRAM (midpoint)**

